## Original Article

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Post-Evaluation Model using Fuzzy Theory

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  **Abstract**

Companies conduct transactions based on signed contracts. Various contracting methods exist for product distribution, including the Multiple Award Schedule (MAS). MAS can be utilized to ensure the provision of high-quality goods to consumers by fostering competition among suppliers. Once a contract is executed, it is essential to evaluate its content and the quality of the delivered products or services. Therefore, post-evaluation is conducted to determine whether the goods were delivered in accordance with the contract and to assess their quality. Satisfaction assessments, including price and service satisfaction, are commonly used as evaluation methods. In general satisfaction evaluations, qualitative indicators are primarily employed. However, these indicators can be ambiguous when capturing subjective opinions. For instance, satisfaction levels are often categorized using a 5-point scale ranging from "very satisfied" to "very dissatisfied." When data is grouped and summarized in this manner, valuable information may be lost due to ambiguity. To address this issue, this study employs fuzzy theory, which translates imprecise human language into a form that computers can process. By applying the proposed fuzzy reliability function, data can be represented within fuzzy intervals, allowing for comprehensive data management without information loss. In this study, we propose a new post-evaluation model using fuzzy numbers to enhance the accuracy of evaluation results. Specifically, we define two fuzzy reliability functions for post-evaluation and apply two distinct types of fuzzy operations. The first utilizes the function principle, while the second introduces a new fuzzy operation with a fixed spread. Additionally, we conduct data analysis using triangular and trapezoidal fuzzy numbers. In our analysis, sample data were generated for evaluation. One type of data, processed using a one-sided fuzzy reliability function, was categorized on a 7-point Likert scale to assess qualitative aspects such as post-satisfaction. The other type, processed using a two-sided fuzzy reliability function, focused on delivery service evaluations. Since the proposed model can be applied to assessments incorporating quantitative indicators, it is expected to enhance the reliability of evaluation results across various fields. Furthermore, by leveraging fuzzy theory, satisfaction levels can be compared while preserving ambiguous information.

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**Keywords:** post-evaluation model; satisfaction; fuzzy reliability function; Multiple Award Schedule (MAS); fuzzy theory; fuzzy numbers

# Introduction

Several users supply or purchase goods in the procurement market. Therefore, for the smooth distribution of the market, the use of some rules is necessary. Based on the rules at the national level, there are various contracts created for management and supervision among companies. Among the contracts, the Multiple Award Schedule (MAS) has shown the best performance. Therefore, the number of contractors who use MAS is increasing every year. MAS was designed to improve the problems of quality degradation and lack of diversity faced by other similar systems. In addition, it is a consumer-centered contract method. This method is widely used in the United States, Canada, and South Korea ([www.pps.go.kr](http://www.pps.go.kr)) [16,17]. MAS focuses on inducing fair competition and supplying quality goods to consumers by selecting multiple suppliers. In addition, as the e-commerce market grows, the suitability of this method is being embraced in some countries that operate MAS. In South Korea, it is used by the public procurement service (PPS) [16,17]. PPS is an organization within South Korea government that is currently serving as an intermediary for sale and purchasing companies at Korea ON-line e-Procurement System (KONEPS). The names of institutions responsible for government procurement and the evaluation of procured items vary across countries. For example, in Germany, this function is managed by the Federal Ministry for Economic Affairs and Energy, while in Canada, it is overseen by Public Services and Procurement Canada (PSPC). In this study, we focus on the Public Procurement Service (PPS) of South Korea, which operates the Multiple Award Schedule (MAS). The post-evaluation criteria for MAS are categorized into delivery period, quality, satisfaction of demand institutions, service, and contract fulfillment sincerity. Each category is further subdivided, resulting in a total of 11 evaluation indicators ([www.gmas.or.kr](http://www.gmas.or.kr)). The weight assigned to each evaluation criterion varies, and the final score is calculated by applying different weights to the grades assigned to each evaluation indicator. Currently, procurement companies undergo three evaluations: pre-evaluation, post-evaluation, and, in some cases, interim inspections. In the pre-evaluation phase, procurement companies are assessed for disqualification reasons and eligibility to participate in bidding through an eligibility evaluation for contractor selection. During post-evaluation, the PPS assesses the quality, delivery period, and service of procured items twice a year. In the MAS phase 2 competition system, these evaluations influence the company’s future contracts. However, the MAS evaluation process remains insufficient. Among the post-evaluation criteria, some indicators, such as the satisfaction of demand institutions, rely on subjective assessments by personnel rather than quantitative measures. If these qualitative indicators are treated the same way as quantitative ones, inaccuracies may arise. Therefore, this study proposes a new post-evaluation model that incorporates fuzzy number-based evaluation index calculations. Several studies have examined post-evaluation models. In [5], Yoon introduced a satisfaction evaluation function for post-supplier assessment in public procurement systems based on MAS, utilizing the reliability function proposed by Derringer and Suich [1]. Recent studies [18-20] have increasingly adopted fuzzy theory to evaluate satisfaction levels. The analysis of causal relationships using fuzzy theory has been the subject of ongoing research [7-15]. This function can take various forms between 0 and 1, depending on the characteristics of the indicator, using either a simple linear function or a general exponential function. The reliability function employed in this study plays a key role in quantifying evaluation results and deriving total scores. The application process determines the current satisfaction level for each indicator through surveys.

While quantitative evaluation indicators rely on direct measurements, qualitative indicators require conversion into quantitative values for statistical analysis. To achieve this, a method is employed where evaluators distinguish satisfaction levels using a scale of 5, 7, or 9, based on the cognitive principle that human short-term memory typically operates within a range of 7±2. Additionally, this study introduces an evaluation method aimed at improving post-evaluation accuracy by addressing the challenges associated with ambiguous data measurement. Studies on satisfaction in supply systems using fuzzy scales were conducted by Kumar et al. in [3,4]. These studies applied the reliability function suggested by Derringer and Suich [1] to express satisfaction evaluation results as function values of fuzzy membership functions. Notably, instead of merely representing satisfaction results as fuzzy membership function values, this study proposes a post-evaluation model that expresses the data itself as a fuzzy number. A fuzzy number is a specialized form of a fuzzy set, introduced by Zadeh [6], that enables the mathematical representation of ambiguous or linguistic expressions. This approach allows for the retention of ambiguous information when analyzing data. For instance, satisfaction is often classified into categories such as "satisfied," "neutral," and "dissatisfied," but these classifications are inherently subjective and difficult to quantify. Evaluator bias can further influence results. By incorporating fuzzy numbers into the post-evaluation model, information loss is minimized, and a more accurate quantification process is achieved. The fuzzy theory addresses ambiguity by using a membership function that represents the degree of belonging to a category. As a result, fuzzy theory has been widely applied in various fields, including regression, mediation analysis, time series analysis, ANOVA (analysis of variance), and chaos theory, where ambiguous data frequently occur. However, the application of fuzzy theory to post-evaluation models in this field has not been extensively explored. Therefore, this study aims to enhance the accuracy of post-evaluation models. The rest of this paper is structured as follows: Section 2 presents a literature review and an overview of fuzzy theory. Section 3 introduces new fuzzy reliability functions for post-evaluation, including the operations of triangular and trapezoidal fuzzy numbers, and explains how post-satisfaction is calculated using a fuzzy scale. Section 4 discusses data analysis using the fuzzy post-evaluation model. Finally, conclusions are drawn in Section 5.

**2. Fuzzy numbers**

Recently, research on artificial intelligence (AI), which functions similarly to the human brain, has been actively conducted. For AI to accurately understand and fulfill human needs, it must be capable of processing all languages used by humans. Human language includes precise expressions, such as "two" and "10 degrees," but also ambiguous expressions, such as "a few" and "about 10 degrees," which can be challenging to interpret. Fuzzy theory provides a theoretical framework for mathematically addressing these ambiguities in human language.

This theory, first introduced by Zadeh [6], plays a key role in minimizing information loss when expressing imprecise data. Zadeh initially derived fuzzy theory from the concept of how the degree of beauty can be expressed.

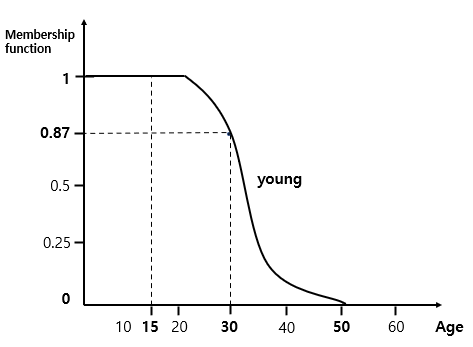
One of the evaluation indicators in post-evaluation models, 'consumer satisfaction,' is also measured using vaguely expressed data. Therefore, to obtain more accurate post-evaluation results, it is necessary to express and measure evaluation indicators using fuzzy theory.

To illustrate a fuzzy set, consider the concept of ‘young age.’ Generally, ‘young age’ is subjective and can encompass various ranges, such as 0 to 30 years or 0 to 40 years, depending on individual perception.

In classical set theory, ‘young age’ does not constitute a set because the degree of membership is unclear. However, a fuzzy set can express this ambiguity by assigning a degree of membership. For example, many people consider individuals between the ages of 20 and 30 as young, but 20 is more definitively young than 30. Simply categorizing people as ‘young’ is often insufficient; in some cases, it is necessary to distinguish different levels of youth. Here, the fuzzy set assigns a degree to these distinctions.

Suppose that the membership function of the fuzzy set representing ‘young age’ is illustrated in Figure 1. The value of the membership function indicates the degree of belonging to the fuzzy set. As shown in Figure 1, a 15-year-old person has a membership value of 1 in the ‘young age’ category, a 30-year-old has a value of 0.87, and a person aged 55 or older has a value of zero.

A fuzzy number is a special type of fuzzy set, which is defined as follows:



**Figure 1.** Fuzzy set ‘young age’

**Definition 1.** Fuzzy number

For the fuzzy membership function of a fuzzy set *,*  is a fuzzy number if satisfies the following properties:

(Normality) There exist , which satisfies

(Fuzzy Convexity) For particular , and

if ,

(Upper semi-continuity) For any ,

then is a fuzzy number

|  |  |
| --- | --- |
|  |  |

**Figure 2.** Membership functions for triangular and trapezoidal fuzzy numbers

|  |  |
| --- | --- |
|  |  |

**Figure 3**. Various types of fuzzy numbers with extended triangular and trapezoidal fuzzy numbers

There are many types of operations of fuzzy numbers. The operations based on Zadeh’s extension Principle [6] is widely used. The extension principle is expressed as follows:

Suppose that is a cartesian product, is a fuzzy set in , respectively, and is a mapping, then the extension principle allows us to define a fuzzy set in as follows:

In the case of , the extension principle reduces to a fuzzy set defined by

There are numerous forms of fuzzy number. Among them, the most commonly used is triangular fuzzy number, and is easy to use because it can be expressed in three points. In addition, there are various types of triangular and trapezoidal fuzzy numbers as shown in Figure 2.

The triangular and trapezoidal fuzzy numbers are denoted by and respectively. In addition, they can be represented by using spreads as follows:

where

and (

where

In this study, we employed triangular fuzzy numbers and trapezoidal fuzzy numbers.

When the satisfaction was measured highest at a particular value, triangular fuzzy number was employed. (see Figure 2.) Conversely, when measuring the satisfaction of each evaluation item measured by the survey with fuzzy data, trapezoidal fuzzy number was employed. (see Figure 2.) Although the triangular fuzzy number is most commonly used in fuzzy data analysis, the trapezoidal fuzzy number is the most appropriate as a fuzzy scale that expresses a person’s mind. This is because it is better for the point indicating the largest degree of membership degree to be expressed in a range rather than a single number.

3. **Post-evaluation function for fuzzy data**

*3.1. Proposed Fuzzy Reliability Function for Post- evaluation of Each Item*

*3.1.1 Proposed One-sided Fuzzy Reliability Function*

Here, we consider the satisfaction evaluation function applicable to fuzzy data using the reliability function proposed by Derringer and Suich [1] introduced.

Suppose the total number of companies to be surveyed is, and the total number of items evaluated is .

If is the satisfaction evaluation function for -th item of the -th company, it is expressed as follows.

(1)

where

fuzzy satisfaction with -th item of the -th company,

minimum fuzzy satisfaction with -th item, and

maximum fuzzy satisfaction with -th item.

This is an indicator that normalizes the fuzzy satisfaction (or fuzzy score) measured by the fuzzy scale and evaluates it on the same basis regardless of unit or size.

*3.1.2 Proposed Two-sided Fuzzy Reliability Function.*

When the response has a specific objective value for evaluating the satisfaction, the two-sided fuzzy reliability function is used. The fuzzy reliability function for evaluating satisfaction obtained by fuzzy data is expressed as:

(2)

fuzzy satisfaction with -th item of the -th company

minimum fuzzy satisfaction with -th item

maximum fuzzy satisfaction with -th

: specific objective value for the -th item of the -th company

*3.2. Proposed operations for trapezoidal fuzzy numbers*

Typically, the operation of the fuzzy number is calculated using Zadeh’s Extension Principle [6]. However, this method increases complexity in multiplying or dividing operations, and results in many cases. Therefore, in this study, we proposed an operation that transforms the interval operation in the calculation of the satisfaction evaluation function. The basic fuzzy operation has crucial drawback that if we repeat the operations then the spreads of fuzzy numbers increase more than those of their original forms. Therefore, we employed two different fuzzy operations. Definition 2 is the fuzzy operations using function principle by Gani [2]. Definition 3 is the proposed new operations using fixed spreads.

**Definition 2. (**Operation of Triangular and trapezoidal fuzzy number using function principle**)**

The following four operations can be performed on triangular fuzzy numbers: Let ,

1) Addition:

2) Subtraction:

3) Multiplication:

4) Division:

Similarly, the following four operations can be performed on trapezoidal fuzzy numbers:

Let ,

1) Addition:

2) Subtraction:

3) Multiplication:

4) Division:

Next, the new operations using fixed spreads were provided.

**Definition 3. (**New operations for symmetric fuzzy numbers**)**

Let be two triangular fuzzy numbers, where , then fuzzy operations for symmetric triangular fuzzy numbers with fixed spreads are expressed as follows:

Moreover, the following four operations can be performed on symmetric trapezoidal fuzzy numbers.

Let be two trapezoidal fuzzy numbers, where . Then fuzzy operations for symmetric trapezoidal fuzzy numbers with fixed spreads are expressed as follows:

where

**Remark.** (Modification of fuzzy operations)

For the case when the left or right end points are not located in the defined domain, they are modified as follows:

If then the result is defined by

If  *,* then the result is defined by

*3.3. Post-evaluation function using fuzzy scale*

In this section, we propose a Post-evaluation function using a fuzzy scale in which the result of fuzzy satisfaction evaluation of each item is fused into one using the above calculation. The weight considering the importance of item -th of the-th company is defined as and the post-evaluation function for each company is defined as follows:

where, (3)

where

# 4. Subsection Data Analysis using Fuzzy Post-Evaluation Model for PPS of South Korea

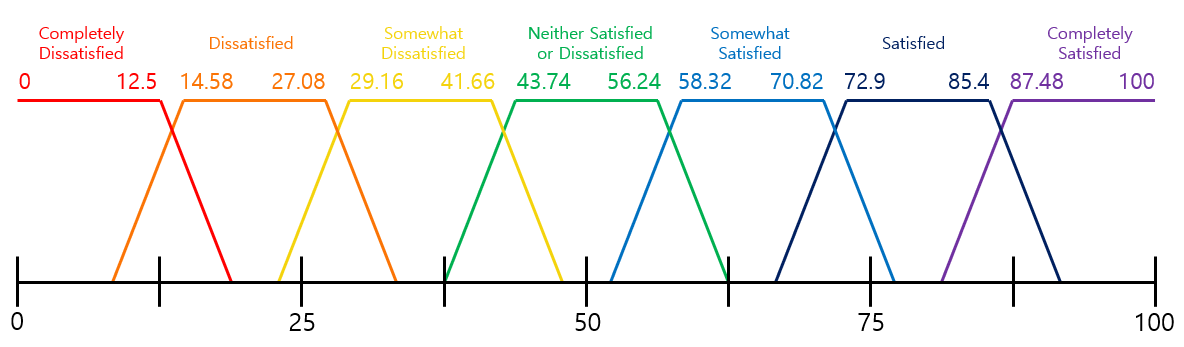
# Fuzzy Post-Evaluation Model for PPS of South Korea using one-sided fuzzy reliability function

First, the data from PPS (https://data.g2b.go.kr/) were employed to check the status of MAS in South Korea, as shown in Figure 4 [16,17].

**Figure 4.** Status of MAS in South Korea

**Table 1 .** Sample data for MAS

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Company Name | Delivery Observation | Delivery Delay | Defect Care | Quality Sat. | Price Sat. | Service Sat. | Post- Sat. | Supply Ratio | Defect Care Period | Fraud Penalty | Trade Penalty |
| A | 75 | 10 | 5 | 5 | 4 | 3 | 3 | 76 | 9 | 30 | 45 |
| B | 90 | 4 | 1 | 1 | 5 | 6 | 5 | 90 | 3 | 3 | 6 |
| C | 35 | 20 | 8 | 8 | 1 | 0 | 1 | 43 | 15 | 120 | 130 |
| D | 82 | 7 | 2 | 2 | 5 | 4 | 4 | 87 | 6 | 15 | 30 |
| E | 45 | 17 | 7 | 7 | 3 | 2 | 2 | 50 | 12 | 90 | 80 |



**Figure 5.** Satisfaction score using trapezoidal fuzzy number

Thereafter, to evaluate the performance of the proposed model, we generated 5 sample data on Table 1. Evaluation categories and allocation weights were the same as the current MAS system (http://www.gmas.or.kr). We used 7 point Likert scale for Qualitative categories like ‘Quality satisfaction’, ‘Price satisfaction’, ‘Service satisfaction’, and ‘Post-satisfaction’, and they were surveyed as percentages of satisfaction. Thereafter, we fuzzified the response shown in Figure 5 to calculate post-evaluation score.

In addition, each quantitative category was considered as an interval instead of one number for interval operation with fuzzy numbers. For example, the crisp value 10 was fuzzified by trapezoidal fuzzy number (10, 10, 10, 10). Data of each category was converted, and then calculated as a score between 0 and 1. Finally, we obtained post-evaluation score, weighted mean of those scores using the allocation points. We employed one-sided fuzzy reliability function (1) and post-evaluation function (3) to evaluate the companies. To calculate post-evaluation score, fuzzy operation from Definitions 2 and 3 were applied. In the process of the operation, a value might be less than 0 or greater than 1. Thus, a **Remark** is provided to correct it because the value should be in (0, 1).

Post-evaluation scores of 5 sample data using function principle of Definition 2 are listed in Table 2, and are shown as a fuzzy membership functions in Figure 6. Consequently, the score was shown in the order of companies B, D, A, E and C. The fuzzy post-evaluation score of company B after correction is (0.848, 1.000, 1.000, 1.000), which is the highest score, while that of company C is (0.000, 0.000, 0.046, 0.171), which the lowest score. The result of new operations of Definition 3 is in Table 3 and Figure 7. The rank of the companies is the same as the previous one using the function principle. Company B, one with the highest score, receives (0.845, 0.907, 1.000, 1.000). The score of company C is (0.000, 0.000, 0.046, 0.171), which is the highest score among the 5 companies.

The spreads of the score with function principle becomes wider than those with new operations. However, as shown in Tables 2 and 3, it can be observed that the sizes of spreads are reduced when using operations with fixed spread. Thus, it can be used to achieve more precise comparison.

Moreover, if the analysis was conducted without fuzzy theory, the scores would be a single number, and would not reflect uncertainty and vagueness of the data, resulting in loss of information.

**Table 2**. Post-evaluation score of 5 companies with function principle.

|  |  |  |
| --- | --- | --- |
| Company Name | Fuzzy Post-evaluation Score | |
| Before correction | After correction |
| A | (0.502, 0.600, 0.645, 0.903) | (0.502, 0.600, 0.645, 0.903) |
| B | (0.848, 1.000, 1.000, 1.326) | (0.848, 1.000, 1.000, 1.000) |
| C | (-0.171, -0.068, 0.046, 0.171) | (0.000, 0.000, 0.046, 0.171) |
| D | (0.692, 0.820, 0.841, 1.148) | (0.692, 0.820, 0.841, 1.000) |
| E | (0.117, 0.199, 0.278, 0.471) | (0.117, 0.199, 0.278, 0.471) |

|  |  |
| --- | --- |
|  |  |

**Figure 6.** Fuzzy post-evaluation score with function principle before correction(left) and the same one after correction(right)

**Table 3.** Post-evaluation score of 5 companies with new operations.

|  |  |  |
| --- | --- | --- |
| Company Name | Fuzzy Post-evaluation Score | |
| Before correction | After correction |
| A | (0.490, 0.553, 0.735, 0.797) | (0.490, 0.553, 0.735, 0.797) |
| B | (0.845, 0.907, 1.135, 1.198) | (0.845, 0.907, 1.000, 1.000) |
| C | (-0.130, -0.068, 0.068, 0.130) | (0.000, 0.000, 0.068, 0.130) |
| D | (0.686, 0.748, 0.955, 1.017) | (0.686, 0.748, 0.955, 1.000) |
| E | (0.123, 0.186, 0.334, 0.397) | (0.123, 0.186, 0.334, 0.397) |

|  |  |
| --- | --- |
|  |  |

**Figure 7.** Fuzzy post-evaluation score with new operations before correction(left) and the same one after correction(right)

*4.2. Fuzzy Post-Evaluation Model for delivery service using two-sided fuzzy reliability function*

Let us consider a delivery system. When a customer orders some items to be delivered at a specific time, then if delivery is made earlier or later than that time, customer satisfaction will be decreased. Therefore, in this case, it is reasonable to use the two-sided fuzzy reliability function introduced above. For data analysis, we generated the following fuzzy data.

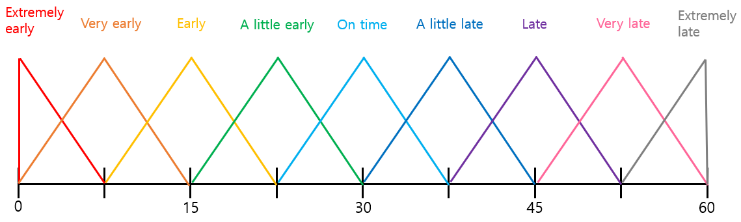
A sample data is proposed in Table 4 to analyze fuzzy post-evaluation model using two-sided fuzzy reliability function. Table 4 shows a sample data for delivery service of 5 stores.

When the customer ordered some items to be delivered at 12:30 pm, if it is delivered earlier then 12:30 or later than 12:30, then the customer will not be satisfied with this delivery service. Let us consider only the time interval from 12:00 pm to 13:00 pm. Therefore, the time interval that we have considered is [0, 60] based on minutes.

Because the scores in Table 4 are based on the arrival time, it is clear that a store can obtain the highest score when they deliver some items exactly at 12:30 pm; thus, it is better to use triangular fuzzy numbers than trapezoidal fuzzy numbers in this case. Using the interval [0, 60], the fuzzified arrival time is expressed in Figure 8. The arrival time is fuzzified using triangular fuzzy numbers with spread 7.5.

**Table 4**. Sample Data for Delivery Service

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Store | | ArrivalTime1 | Score 1 | ArrivalTime 2 | Score 2 | ArrivalTime3 | Score 3 | ArrivalTime 4 | Score 4 | ArrivalTime 5 | Score 5 |
| A | 11:50 | | 20 | 11:42 | 12 | 12:24 | 54 | 12:11 | 41 | 11:45 | 15 |
| B | 12:01 | | 31 | 12:03 | 33 | 11:54 | 24 | 12:08 | 38 | 11:57 | 27 |
| C | 12:24 | | 54 | 12:30 | 60 | 11:31 | 1 | 11:33 | 3 | 12:29 | 59 |
| D | 11:47 | | 17 | 11:52 | 22 | 12:01 | 31 | 11:58 | 28 | 12:11 | 41 |
| E | 12:14 | | 44 | 11:39 | 9 | 11:30 | 0 | 12:28 | 58 | 11:54 | 24 |

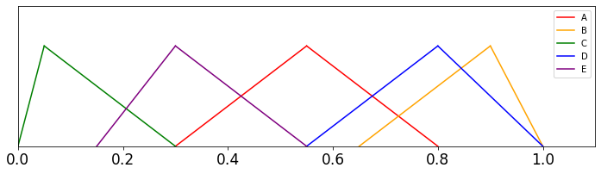


**Figure 8.** Post-Evaluation score using triangular fuzzy numbers

The fuzzy post-evaluation score of the given data using (2) and (3) is provided in Table 5. To avoid having large spread, the function principle in Definition 2 was applied.

**Table 5**. Fuzzy post-evaluation scores of the delivery service data calculated by function principle

|  |  |
| --- | --- |
| Store | Fuzzy post-evaluation score |
| A | (0.3, 0.55, 0.8) |
| B | (0.65, 0.9, 1.0) |
| C | (0.0, 0.05, 0.3) |
| D | (0.55, 0.8, 1.0) |
| E | (0.15, 0.3, 0.55) |
|  |  |



**Figure 9**. Fuzzy post-evaluation scores of the delivery service data calculated by Definition 2

Figure 9 shows the comparison of each fuzzy post-evaluation scores. B shows the best score, and C shows the lowest score. In addition, the fuzzified data are a symmetric triangular fuzzy number; however, using function principle they can be expressed as an asymmetric triangular fuzzy number, as shown in Figure 9. This result shows that the fuzzy post-evaluation score better shows the properties of the particular data, considering that the delivery service of each store is not always the same, and some differences can occur depending on the situations.

The other fuzzy post-evaluation score of the provided data using (2) and (3) is listed in Table 6. The newly defined operations method, Definition 3, was applied.

Figure 10 shows similar results compared to Figure 9. However, in this case, fuzzy post-evaluation scores are also represented by symmetric triangular fuzzy numbers using a fixed spread considering post evaluation score. As an exception, Store C is not symmetric because the fuzzy post-valuation score is (-0.052, 0.073, 0.198), and is adjusted by **Remark** to a value between 0 and 1.

Considering that the spread of fuzzy post-evaluation scores is narrower in Figure 10, it can be observed that the New operations is more helpful method than function principle, which evaluates the delivery service of each score when comparing data.

To summarize the results of the analysis by function principle (Definition 2) and new operations (Definition 3), function principle has the disadvantage that the degree of ambiguity is considerably expressed; thus, it cannot be distinguished between each company. Accordingly, it has trouble in differentiating and evaluating companies.

Conversely, when using new operations (Definition 3), it can be observed that the spread ranges are narrower than those of the results by function principle (Definition 2). When using this definition, discrimination between companies is differentiated between companies, allowing us to determine the best company.

# 5. Conclusions

There are several contracting methods for product distribution. In general, the data used in these contract methods exhibit slight vagueness and ambiguity. Fuzzy theory is commonly used to manage such ambiguous data. Therefore, we employed fuzzy numbers to represent vague information in our data analysis. In this study, a fuzzy post-evaluation function was proposed. One-sided and two-sided fuzzy reliability functions were defined. The reliability functions were applied differently, depending on whether the best characteristic was derived from large or nominal data. Using these fuzzy reliability functions, each item was evaluated. The post-evaluation function was calculated as a weighted average of the fuzzy reliability function values, where each weight reflected the importance of the corresponding item. For data analysis, two simulated datasets were provided for the one-sided and two-sided fuzzy reliability functions. In calculating these functions, two different operations were applied. The first was based on the function principle, which is widely used. However, this operation has a drawback: it tends to produce results that are broader than expected. To address this issue, fuzzy operations using fixed spreads were proposed to obtain more reasonable spreads. According to our data analysis, the newly defined fuzzy operations proved to be more effective for data comparison than existing fuzzy operations. In this study, we applied the post-evaluation model only to sampled data, including MAS data. However, it has the potential to be applied in various situations requiring a post-evaluation system or satisfaction survey. Therefore, in our future research, we plan to explore further applications incorporating diverse measurement methods to evaluate satisfaction for quality control using fuzzy theory.

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